Confinement of Alpha Particles in a Low-Aspect-Ratio Tokamak Reactor

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**Introduction**

Important features of spherical tokamak (ST) reactors

1. **High elongation**
2. **High triangularity**
3. **Low aspect ratio**
   - improvement of level of symmetry
   - line symmetry $\Rightarrow$ point symmetry
4. **Non-inductive plasma current** (hollow current profile)
   - no use of center solenoid
   - improvement of plasma performance
   - negative shear
   - access to the 2\textsuperscript{nd} stability region

Confinement of alpha particles in an ST reactor is very interesting.
- Neoclassical confinement (ripple loss),
- Non-neoclassical confinement (TAE mode etc.)

Here we focus our attention only on the neoclassical confinement.
Introduction

Objectives

- to investigate the confinement of alpha particles in a low-aspect-ratio tokamak by using an orbit-following Monte-Carlo code.

Preliminary results indicate that the ripple loss of alpha particles shows a marked reduction in a low-aspect-ratio system.

The number of toroidal field (TF) coils is one of the key parameters for the design of tokamak system. The smaller the number of TF coils is, the easier the design of reactor system becomes.

Preliminary results also show that in a tokamak system with a conventional aspect ratio, the reduction of the number of TF coils results in a considerable increase of alpha particle losses.

Is there a possibility to reduce the number of TF coils in a low-aspect-ratio tokamak?

- to investigate the possibility to design a tokamak system with a low aspect ratio and a low number of TF coils.
Neoclassical confinement of $\alpha$ particles $\equiv$ ripple loss of $\alpha$ particles

Ripple losses
- ripple-trapped loss
  - collisional trapping
  - collisionless trapping in a non-uniform ripple distribution (a)
- ripple-enhanced banana drift (b)

If the drift exceeds a critical value, the trajectory becomes stochastic
[Goldston, White and Boozer (1981)].
Ripple-loss processes of suprathermal alpha particles

- Model MHD equilibrium (uniform ellipticity $\kappa$ and triangularity $\Delta$)

\[
\psi = \psi (\rho)
\]

\[
\rho^2 = \left[ 1 + \frac{\Delta}{\alpha} (R - R_t) \right] \frac{Z_c^2}{\kappa^2} + (R - R_t)^2
\]

$a$ : minor radius, $R_t$ : major radius

\[
Z = \frac{\kappa}{\sqrt{1 + \Delta (R - R_t) / a}} Z_c
\]

\[
B_R \approx \sqrt{1 + \Delta (R - R_t) / a} B_R^c
\]

Hereafter, variables with the superscript ‘c’ denote those in a circular plasma with the same $q_s(a)$.

1) ripple-well parameter

\[
\alpha \equiv \frac{1}{N \delta} \left| \frac{B_R}{B} \right| \equiv \sqrt{1 + \Delta (R - R_t) / a} \alpha^c
\]

$N$ : number of toroidal field coil,

$\delta$ : local field ripple

Ripple wells are developed in the region $|\alpha| < 1.0$. 
2) ripple-enhanced banana drift

\[
P^* \phi = Z^* \frac{\partial \psi}{\partial Z} \equiv \sqrt{N \pi} \left| \frac{B_R}{B} \right|^{-3/2} \rho_L \delta \frac{\partial \psi}{\partial Z}
\]

\[
P^* \phi \approx \frac{1}{\kappa \left[ 1 + \Delta (R - R_t) / a \right]^{1/4}} \frac{P^* \phi c}{\psi c}
\]

\(Z^*: Z\) displacement enhanced by ripple, \(\rho_L:\) Larmor radius

3) critical field ripple for stochastic orbits

\[
\frac{d \varphi_b}{d \psi} P^* \phi \geq \frac{1}{N} \quad \varphi_b: \text{toroidal angle difference between two banana tips}
\]

\[
\delta_s = \kappa \left[ 1 + \frac{\Delta}{a} \left\{ \frac{1}{4} (R - R_t) + \frac{1}{\pi} (\rho + q_s / q'_s) \right\} \right] \delta^c_s
\]

\[
\delta^c_s \approx \frac{1}{(N \pi R_t q_s / \rho)^{3/2} \rho_L q'_s} \quad (\text{By Goldston-White-Boozer})
\]

Banana orbits in the region \(\delta > \delta_s\) become stochastic.
Ripple-loss processes of suprathermal alpha particles

Summary of dependence of
- ripple-well parameter \( \alpha \),
- ripple-enhanced banana drift \( P_\phi^* \) and
- critical field ripple of stochastic orbit \( \delta_s \)
on some important system parameters

1) dependence on the elongation \( \kappa \) (Tani et.al in Nucl.Fusion1993)

\[
\alpha \propto \text{independent} , \quad P_\phi^* \propto 1 / \kappa , \quad \delta_s \propto \kappa
\]

2) dependence on the aspect ratio \( A \)

\[
\alpha \propto 1 / A , \quad P_\phi^* \propto A^{3/2} , \quad \delta_s \propto 1 / A^{3/2}
\]

3) dependence on the local field ripple \( \delta \) and the number of TF coils \( N \)

\[
\alpha \propto 1 / (N \delta) , \quad P_\phi^* \propto N^{1/2} \delta , \quad \delta_s \propto 1 / N^{3/2}
\]
Simulation studies on the ripple loss of alpha particles by using an orbit-following Monte-Carlo code

(1) Qualitative investigations of the ripple loss using analytical MHD equilibria into
   1-1) the dependence of the aspect-ratio
   1-2) the dependence of ripple losses on the number of TF coils

(2) Quantitative investigations of the ripple loss using a realistic MHD equilibrium of VECTOR into
   2-1) the dependence on the edge field ripple for a hollow and a parabolic plasma current profile
   2-2) the dependence on the number of TF coils for a hollow and a parabolic plasma current profile
   2-3) the allowable field ripple and the number of TF coils
Simulation results on ripple losses of alpha particles

Analytical MHD equilibria

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius</td>
<td>$R_t = 3.7 \sim 9.2 \text{m}$</td>
</tr>
<tr>
<td>Minor radius</td>
<td>$a = 1.9 \text{m}$</td>
</tr>
<tr>
<td>Toroidal field $@R=R_t$</td>
<td>$B_t = 3.1 \text{T}$</td>
</tr>
<tr>
<td>Plasma temperature</td>
<td>$T_e(\Psi) = T_{e0} (1-\Psi)$</td>
</tr>
<tr>
<td></td>
<td>$T_i(\Psi) = T_{i0} (1-\Psi)$</td>
</tr>
<tr>
<td></td>
<td>$T_D(\Psi) = T_T(\Psi) = T_I(\Psi)$</td>
</tr>
<tr>
<td></td>
<td>$T_{e0} = T_{i0} = 35 \text{keV}$</td>
</tr>
<tr>
<td>Plasma density</td>
<td>$n_e(\Psi) = n_{e0} (1-\Psi)^{0.3}$</td>
</tr>
<tr>
<td></td>
<td>$n_D(\Psi) = n_T(\Psi) = n_i(\Psi)$</td>
</tr>
<tr>
<td></td>
<td>$n_{e0} = 2 \times 10^{20} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>Plasma current</td>
<td>$j(\Psi) = j_0 (1-\Psi^{1.3})$</td>
</tr>
<tr>
<td>Safety factor</td>
<td>$q_s = 2.56$</td>
</tr>
<tr>
<td>Elipticity</td>
<td>$\kappa = 1.55$</td>
</tr>
<tr>
<td>Tiangularity</td>
<td>$\Delta = +0.5$</td>
</tr>
<tr>
<td>Effective Z</td>
<td>$Z_{\text{eff}} = 1.9 \text{ (uniform)}$</td>
</tr>
<tr>
<td>Charge number of impurity</td>
<td>$Z_{\text{imp}} = 6.0 \text{ (carbon)}$</td>
</tr>
<tr>
<td>Number of TF coils</td>
<td>$N = 4 \sim 18$</td>
</tr>
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</table>
Simulation results on ripple losses of alpha particles

Distribution of the field ripple in a system with vertically long TF coils

\[
\delta \equiv \delta_o \left( \frac{R}{R_t + a} \right)^N - 1 + \delta_i \left( \frac{R_t - a}{R} \right)^N - 1
\]

\[\delta_o, \delta_i: \text{field ripples at outer and inner plasma edge}\]

The distribution of the field ripple is strongly depends on the aspect ratio.
Simulation results on ripple losses of alpha particles

(1) Qualitative investigations of the ripple loss using analytical MHD equilibria

1-1) dependence on the aspect ratio $A$

\[
\begin{align*}
\alpha & \propto \frac{1}{A}, \\
P^*_\phi & \propto A^{3/2}, \\
\delta_s & \propto \frac{1}{A^{3/2}}
\end{align*}
\]

As the aspect ratio is reduced,

- the area of ripple-well region
- the ripple-enhanced banana drift
- the area of stochastic orbit region

The distribution of the field ripple also strongly depends on $A$.

The ripple loss shows a very strong dependence on $A$ by the synergy of all of these effects.

$q_a = 2.56$
$\delta_o = 1\%$
$N = 12$

$A^{4.3}$
Simulation results on ripple losses of alpha particles

(1) Qualitative investigations of the ripple loss using analytical MHD equilibria

1-2) dependence on the number of TF coils $N$

\[
\alpha \propto \frac{1}{N \delta}, \\
\phi^* \propto N^{1/2} \delta, \\
\delta_s \propto \frac{1}{N^{3/2}}
\]

As the number of TF coils is reduced,
- the area of ripple-well region,
- the ripple-enhanced banana drift,
- the area of stochastic orbit region

Note that the distribution of $\delta$ strongly depends on $N$. 

![Diagram showing the total power loss fraction $G_t$ as a function of $N$.]
Simulation results on ripple losses of alpha particles

Geometry and plasma parameters of VECTOR

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</tr>
<tr>
<td>Number of TF coils</td>
<td>$N = 4 \sim 12$</td>
</tr>
</tbody>
</table>
Simulation results on ripple losses of alpha particles

(2) Quantitative investigations of the ripple loss using a realistic MHD equilibrium of VECTOR

2-1) the dependence on the edge field ripple

Some axi-symmetric loss has been found in a hollow current plasma.
Simulation results on ripple losses of alpha particles

(2) Quantitative investigations of the ripple loss using a realistic MHD equilibrium of VECTOR

2-2) the dependence on the number of TF coils
Simulation results on ripple losses of alpha particles

(2) Quantitative investigations of the ripple loss using a realistic MHD equilibrium of VECTOR

2-3) the allowable field ripple and the number of TF coils (preliminary results using only 4000 test particles)

Allowable heat loads:
\[ \sim 2 \text{Mw/m}^2 \text{ (with cooling system) , } \sim 1 \text{Mw/m}^2 \text{ (w/o cooling system) } \]

<table>
<thead>
<tr>
<th>Current profile</th>
<th>With cooling</th>
<th>Without cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hollow</td>
<td>( \sim 1.5% )</td>
<td>below 1%</td>
</tr>
<tr>
<td>Parabolic</td>
<td>( \sim 2% )</td>
<td>( \sim 1% )</td>
</tr>
</tbody>
</table>

Allowable field ripple assuming a toroidal peaking factor (peak/average over toroidal angle) about 2,

\[ \delta \theta \]

\[ <P_w>_{\text{max}} \frac{(Mw/m^2)}{\text{Toroidal average of heat load}} \]

Diagram showing:
- First wall
- Hitting zone
- Hollow current
- Parabolic current

Graphical representation of heat load and field ripple for hollow and parabolic current profiles with and without cooling.
Simulation results on ripple losses of alpha particles

(preliminary results)

Original VECTOR

N=12, \( \delta_o = 0.5\text{-}1.0\% \)

Low N VECTOR

N=6, \( \delta_o = 1.5\% \text{-} 2.0\% \)

About 30\% increase in the TF-coil size (weight).

A trade-off between the weight of TF coils and the space for blankets, poloidal coils and maintenance.
Conclusions

1. The ripple loss is strongly reduced as the aspect ratio becomes low (proportional to $A^{4.3}$ for $A \sim 3$). Consequently, alpha particles are well confined in a low-aspect-ratio tokamak.

2. In a low-aspect-ratio system, the dependence of the ripple loss on the number of TF coils is very weak, if the edge field ripple is kept constant.

3. Thanks to the good confinement of alphas in a low-aspect-ratio system, the number of TF coils can be reduced to about 6 by making allowances for a small amount of ripple loss and some increase in the size of TF coils.

Future works

1. Optimization of plasma and TF coil shapes to reduce ripple losses of alpha particles.
   
   Our target: a **low A and low N tokamak** without cooling system for loss alphas

2. Improvement of Monte-Carlo error bars for 2D wall heat load by using large number of test particles (>100,000).
Thank you for your attention !!!