Alfvén Eigenmodes in Spherical Tokamaks

S.E.Sharapov, M.P.Gryaznevich, and the MAST Team
Euratom/UKAEA Fusion Association, Culham Science Centre,
Abingdon, Oxfordshire, UK

H.L.Berk
Institute for Fusion Studies, University of Texas at Austin,
Austin, Texas, USA

S.D.Pinches
Max-Plank Institute for Plasmaphysics, Euratom Association,
Garching, Germany

INTRODUCTION

- **The primary motivation for the spherical tokamak (ST) concept is its predicted high-β limit [1]. Record value of volume-averaged $\beta \cong 40\%$ was achieved in START NBI-heated plasmas [2]. The concept of high-β burning plasma STs is considered [3].**
- **Alfvén instabilities are of major concern for magnetic fusion as they can lead to losses/redistribution of fast ions including alpha-particles.**
- **Lots of Alfvén instabilities excited by NBI-produced energetic ions have been observed on START and MAST:**
  - fixed-frequency modes in TAE and EAE frequency range;
  - frequency-sweeping "chirping" modes;
  - fishbones;
  - modes at frequencies above the AE frequency range.
  These instabilities in ST experiments:
  - **provide a test-bed for testing theoretical models on Alfvén instabilities in ITER;**
  - **stimulate experimental studies of energetic-ion-driven instabilities over broad range of plasma beta, up to $\beta(0) \geq 1$ proposed for burning STs [3]**

[1] Y-K M Peng and D J Strickler, Nuclear Fusion 26 (1986) 769
[2] M P Gryaznevich et al., Phys. Rev. Lett. 80 (1998) 3972
[3] H R Wilson et al., Proc. 19th IAEA Fusion Energy Conf. (2002) IAEA-CN-94/FT/1-5

WHY ALFVÉN INSTABILITIES ARE COMMON IN STs?

- **Tight aspect ratio ($R_0 / a \sim 1.2 \div 1.8$) limits the value of magnetic field at level $B_T \sim 0.15 \div 0.6$ in present-day STs ⇒ Alfvén velocity in ST is very low**

$$V_A = B_T / (4\pi n_i m_i)^{1/2} \cong 10^6 \text{ ms}^{-1} \text{ (START)}$$

(compare, e.g. to Joint European Torus (JET), where $V_A \cong 7\times10^6$ ms^{-1})

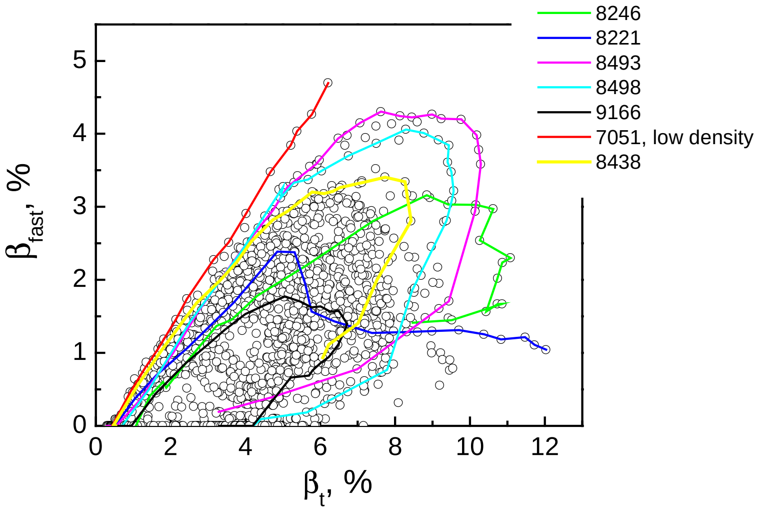
- **Even a relatively low-energy NBI, e.g. 30 keV hydrogen NBI on START had speed**

$$V_{NBI} \cong 2.4\times10^6 \text{ ms}^{-1} > V_A ,$$

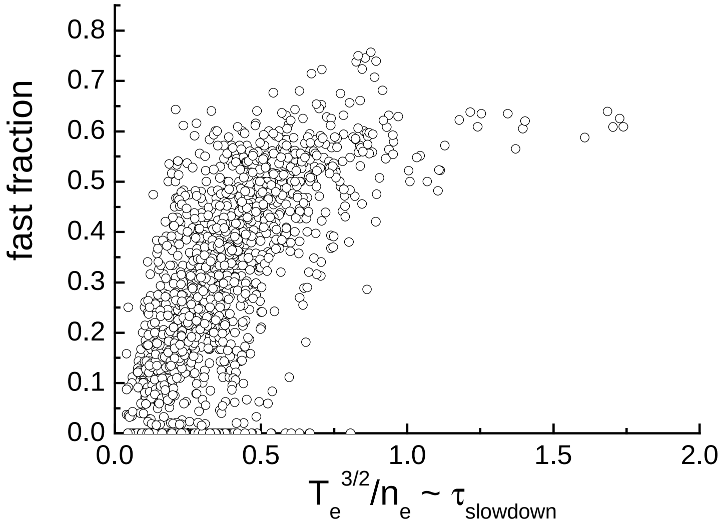
- **The super-Alfvénic NBI can excite Alfvén waves via the fundamental resonance $V_{||NBI} = V_A$. Free energy source for the Alfvén instability: radial gradient of beam ions, $(\gamma/\omega)_{AE} \propto - q^2 r_{AE} (d\beta_{beam}/dr)$**

WIDE RANGE OF PLASMA / BEAM PARAMETERS ON STs

Ratio β_{fast} / $\beta_{thermal}$ in STs can be higher than what is obtained in other tokamaks



Typical values of β_{fast} and $\beta_{thermal}$ in MAST discharges (TRANSP analysis by M.Gryaznevich)



Ratio β_{fast} / $\beta_{thermal}$ vs. slowing-down time in MAST discharges. The spread is caused by difference in NBI power and plasma density.

⇓

both 'perturbative' AEs (TAEs) and 'non-perturbative' Energetic Particle Modes can exist

WIDE RANGE OF PLASMA / BEAM PARAMETERS ON STs

Thermal plasma $\beta_{thermal}$ can be as high as $\beta_{thermal}(0) \sim 1$. High beta can affect Alfvén instabilities in two ways (at least).

1) High plasma pressure suppresses TAEs;

2) Thermal ion Landau damping plays a stronger role. Indeed, since

$$\beta_i \equiv 8\pi n_i T_i / B_T^2 = (2T_i/m_i)\times(4\pi n_i m_i / B_T^2) = (V_{Ti}/V_A)^2$$

Alfvén waves interact stronger with thermal ions as $\beta_{thermal}$ increases. Limiting cases:

low-β discharges:

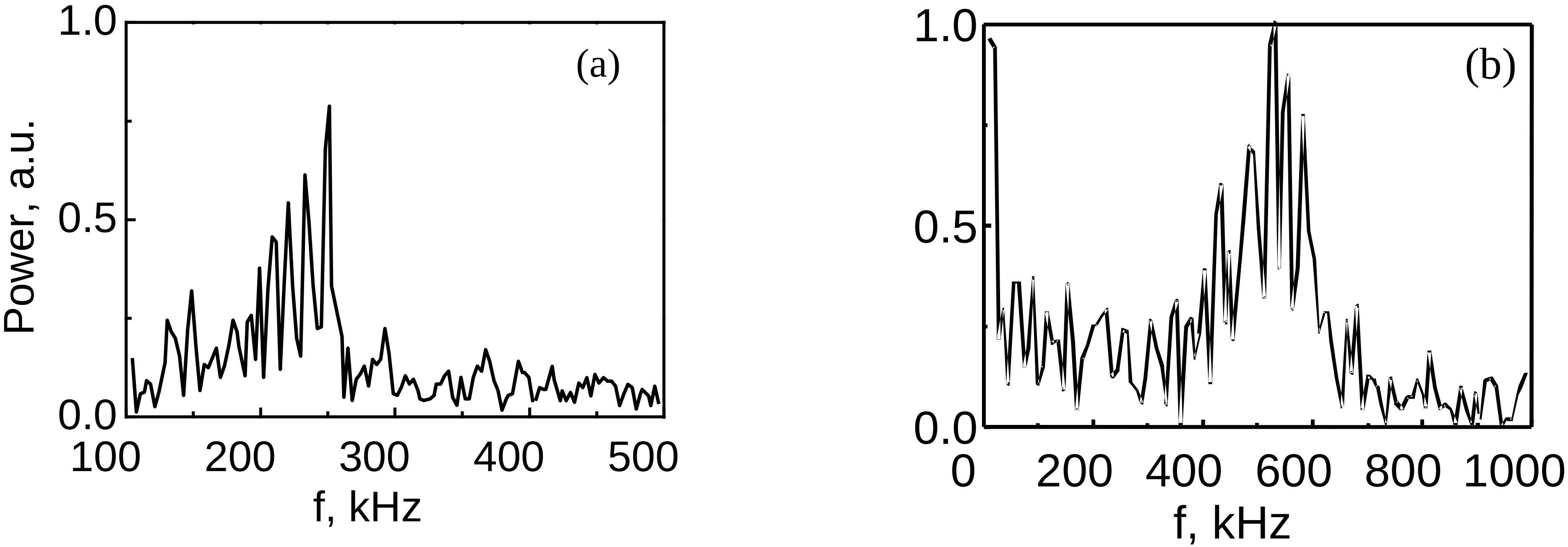
$V_{Ti} << V_A \le V_{beam} << V_{Te}$. Instability is determined by fast ion profile, while thermal ions play a stabilising role (via $V_{||i} = V_A/3$ resonance);

discharges with $\beta_i \sim 1$:

$V_{Ti} \sim V_A << V_{beam} << V_{Te}$. Stability/instability is determined by thermal ions

OBSERVATIONS ON START (LOW-β DISCHARGES)

- START: $R_0 \approx 0.3 \div 0.37$ m; $a \approx 0.23 \div 0.3$ m; $I_P \approx 300$ kA; $B_0 \approx 0.15 \div 0.6$ T
- Hydrogen beam co-injected into D plasmas: $E_{NBI} \cong 30$ keV, $P_{NBI} \leq 0.8$ MW
- **Modes with fixed frequencies** $f_{AE} \cong 200$-250 kHz (#35305), lasting for 1-5 ms, were observed in pulses with $P_{NBI} \leq 0.5$ MW and in early phase of some pulses with $P_{NBI} \leq 0.8$ MW, when $\boldsymbol{\beta_T \leq 3\text{-}5\%}$
- Mode frequency ~ TAE frequency $f_{TAE} \equiv V_A / 4\pi q R_0 \sim 200$ kHz
- Poloidal mode numbers of the excited modes, $m = 1$-4, are in agreement with the strongest drive estimate for TAE, $\boldsymbol{\Delta_{orbit} \sim r_{TAE}/m}$
- Both Toroidal and Elliptical AEs (frequency range $f_{EAE} \approx 2 f_{TAE}$) were observed



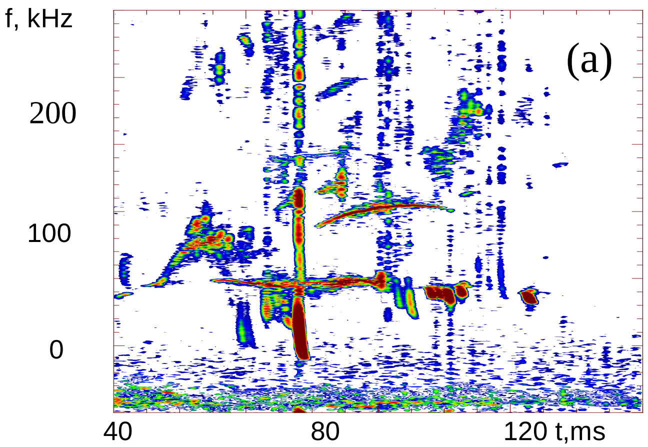
Mirnov coil signal Fourier power spectra of:

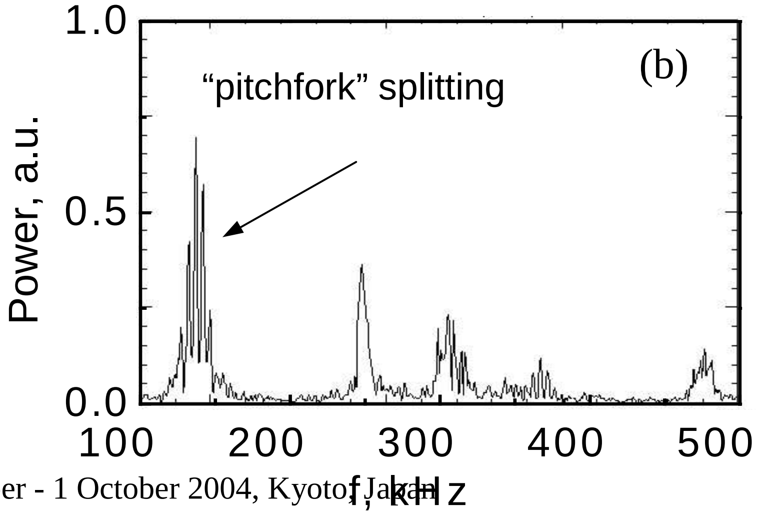
(a) fixed-frequency TAE at t ~ 26ms, START, shot #35305, $\beta < 3\%$;

(b) fixed-frequency EAEs in the EAE gap, t ~ 26.7ms, START #36484, $\beta \sim 4\%$.

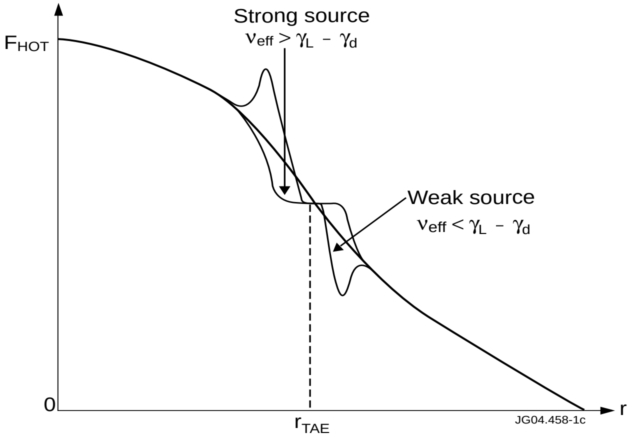
OBSERVATIONS ON MAST (LOW-β DISCHARGES)

- MAST: $R_0 \approx 0.9$ m; $a \approx 0.7$ m; $I_P \approx 1.35$ MA (achieved in 2003); $B_0 \approx 0.4 \div 0.7$ T;
- D beam co-injected into D plasmas: $E_{NBI} \cong 45$ keV, $P_{NBI} \leq 3.2$ MW
- Both TAE and EAE observed on MAST, but the modes are longer lasting (>20 ms), more numerous, with a broader range of unstable n's. Fine **"pitchfork" splitting** of the spectrum is often observed (as shown in the Figure (b) for MAST discharge #2884).



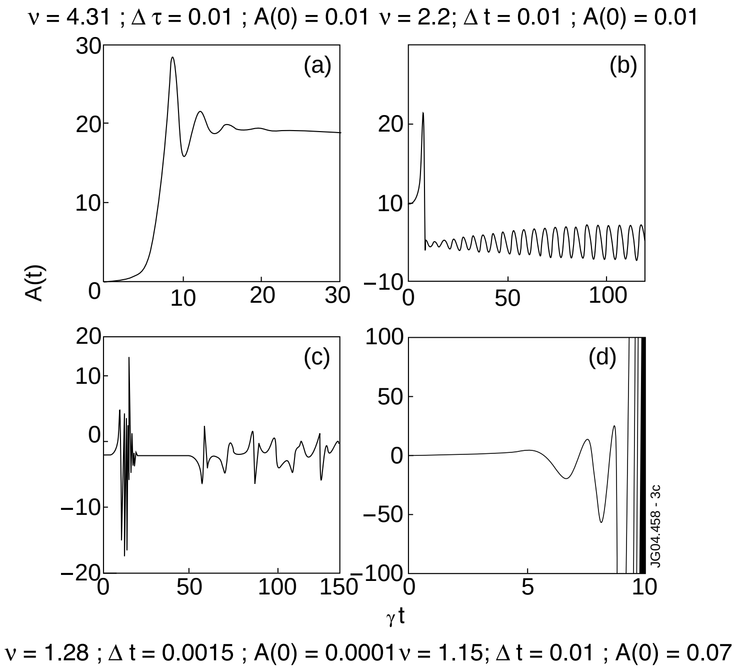


NONLINEAR EVOLUTION OF TAE INSTABILITY



Non-linear TAE behaviour depends on competition between the field of the mode that tends to flatten distribution function near the resonance (effect proportional to the net growth rate $\gamma \equiv \gamma_L - \gamma_d$) and the collision-like processes that constantly replenish it (proportional to ν_{eff})

NONLINEAR EVOLUTION OF TAE INSTABILITY



Nonlinear equation for TAE amplitude

$$\frac{dA}{dt} = A - \exp(i\phi)\int_0^{t/2}\tau^2\int_0^{t-2\tau}\exp\left[-\nu^3\tau^2\left(2\tau/3+\tau_1\right)\right]$$

$$\times A(t-\tau)A(t-\tau-\tau_1)A^*\left(t-2\tau-\tau_1\right)d\tau_1 d\tau$$

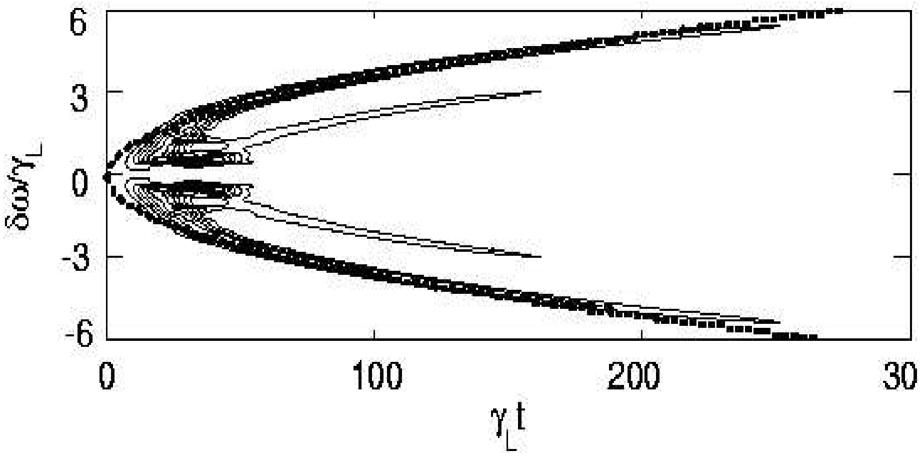
derived in [4] describes four different regimes of TAE:

a) Steady-state (observed);
b) Periodically modulated (observed as ‘pitchfork-splitting’ effect);
c) Chaotic;
d) Explosive regimes of TAE-behaviour as functions of $\nu\equiv\nu_{eff}/\gamma$

- Explosive regime in a more complete non-linear model [5] leads to **frequency-sweeping ‘holes’ and ‘clumps’** on the perturbed distribution function.

[4] H.L.Berk, B.N.Breizman, and M.S.Pekker, Plasma Phys. Reports 23 (1997) 778
[5] H.L.Berk, B.N.Breizman, and N.V.Petviashvili, Phys. Lett. A234 (1997) 213

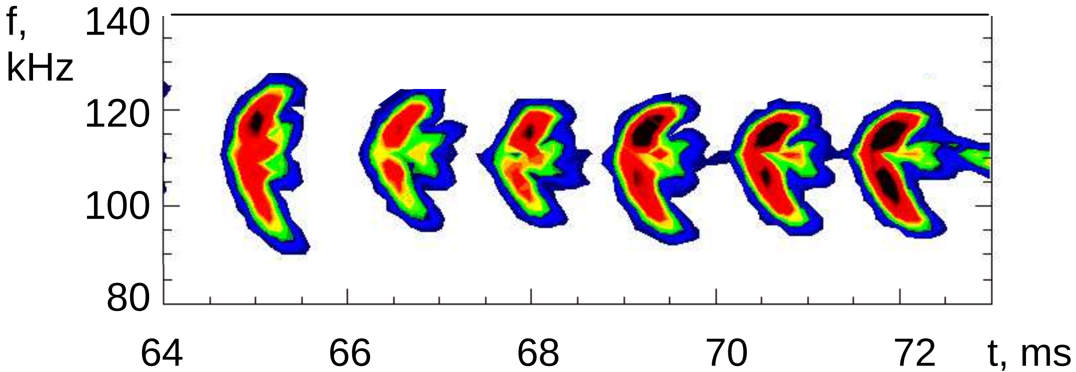
ON THE HOLES AND CLUMPS THEORY

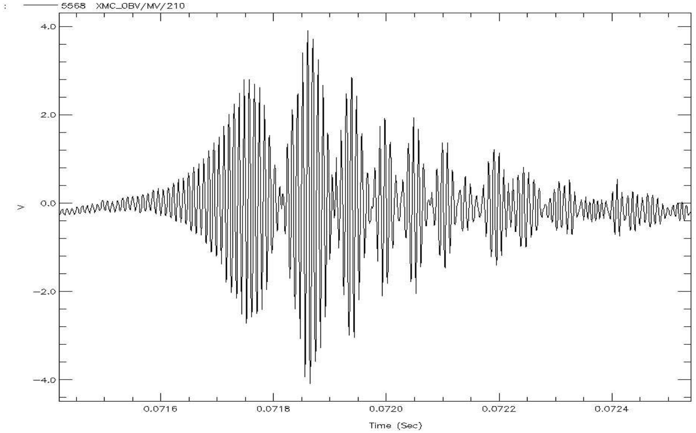


- **Beyond the 'explosive' regime, theoretical prediction shows two long-living thermal fluctuations on the perturbed distribution function.**
- **These long-living Bernstein-Greene-Kruskal (BGK) nonlinear waves sweep in frequency away from the starting frequency, with frequency sweep related to the particle trapping frequency in the TAE field:**

$$\delta\omega \propto \omega_b^{3/2} t^{1/2}; \quad \omega_b(t) \propto \left|\delta B_{TAE}\right|^{1/2}$$

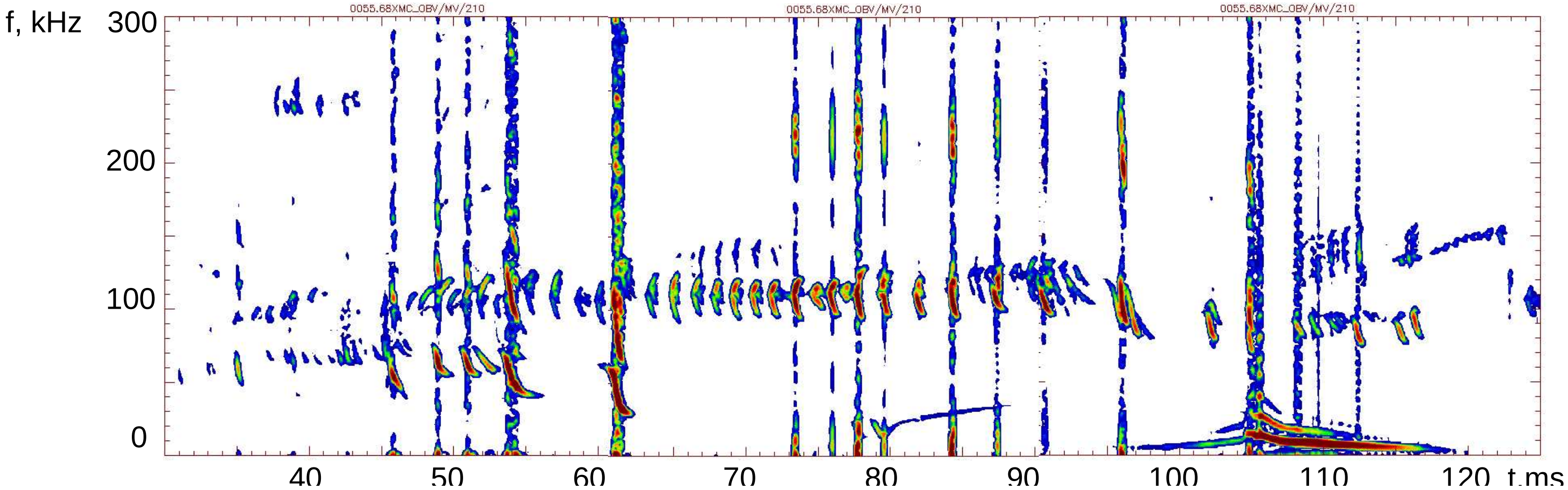
MAST: FREQUENCY-SWEEPING MODES ARISING FROM TAEs





Primary suspect: hole-clump frequency-sweeping pairs

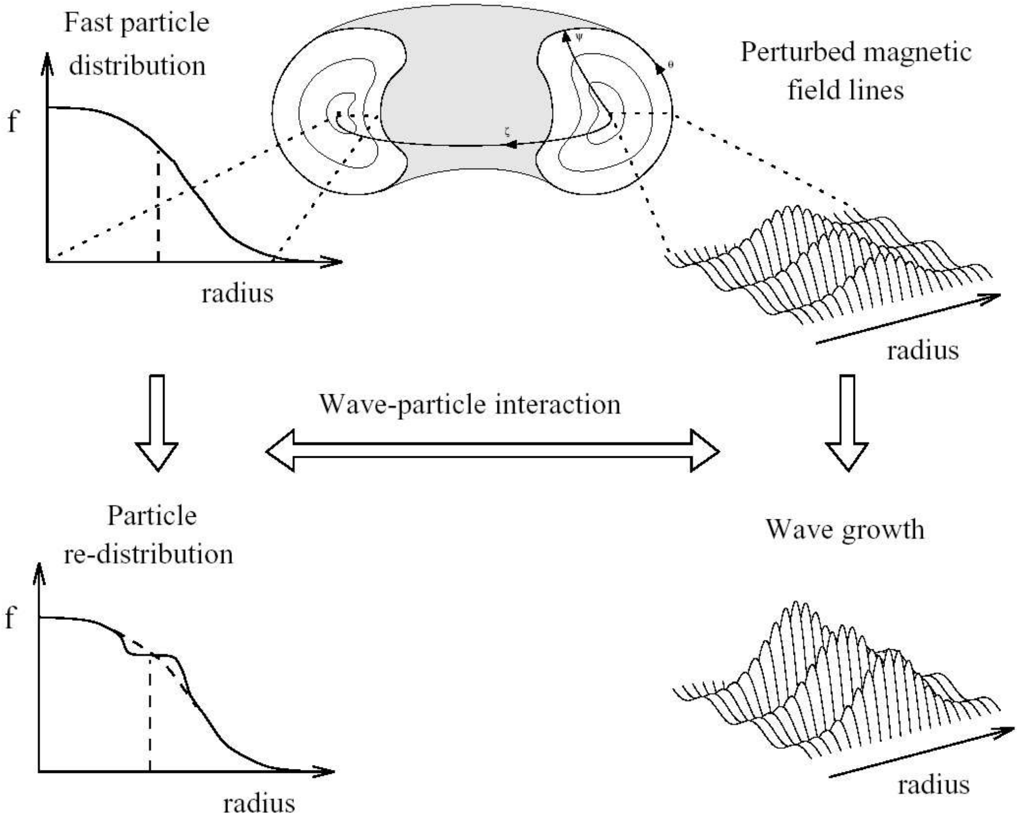
MAST: FREQUENCY-SWEEPING MODES ARISING FROM TAEs



For hole-clump triggering:

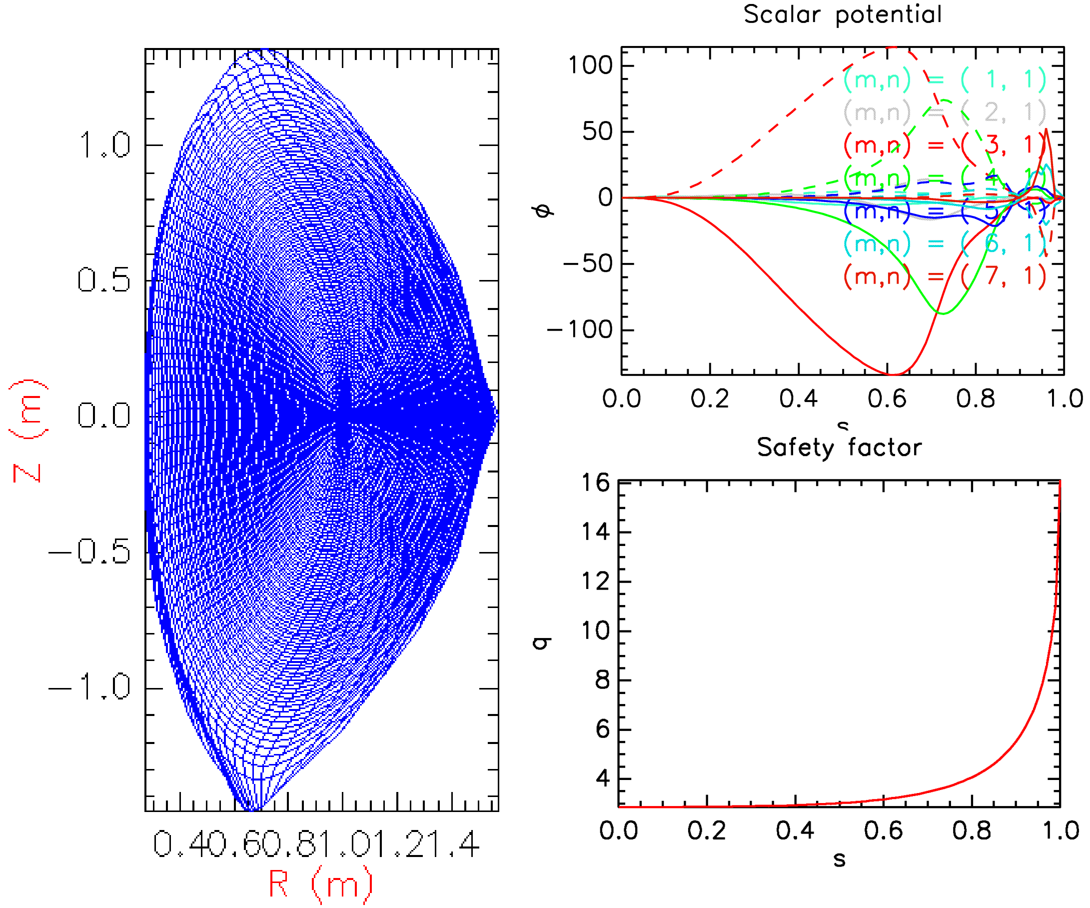
- Plasma should be near the linear instability threshold.
- Collisional effects should be sufficiently weak to allow an “explosive” initialisation of holes and clumps. *This means, that the up-chirping modes are likely to be observed at lower densities or higher temperatures.*

INTERPRETING THE SWEEPING MODES WITH HAGIS CODE[6]

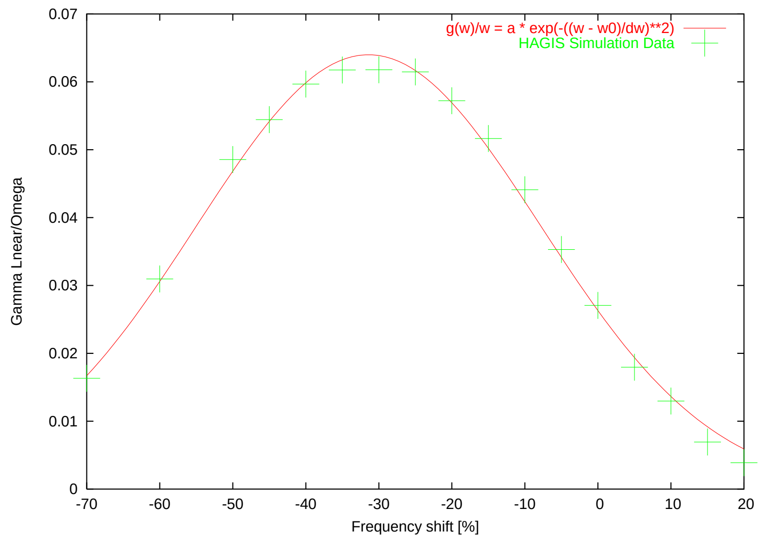


[6] S.D.Pinches et al., Computer Physics Communications 111 (1998) 133

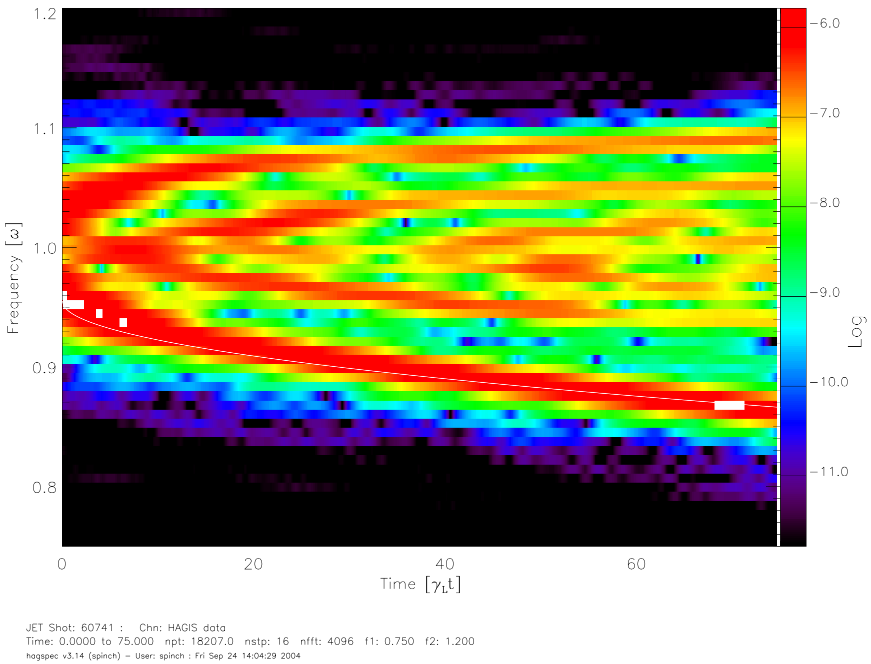
INTERPRETING THE SWEEPING MODES WITH HAGIS CODE



INTERPRETING THE SWEEPING MODES WITH HAGIS CODE



Growth rate as a function of mode frequency ω. Up-down symmetric frequency-sweeping modes are obtained for **ω at the maximum point.**

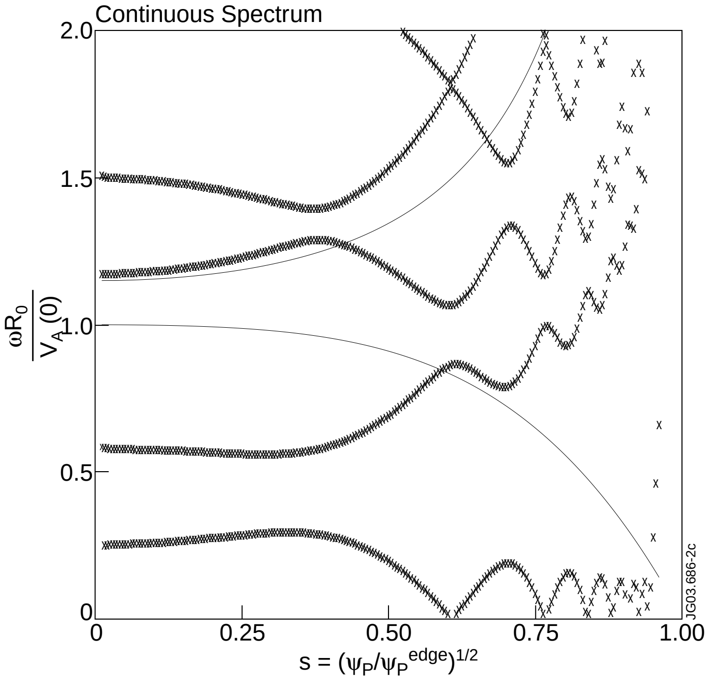


Amplitude of the TAE perturbation as a function of time and frequency. $\gamma_L/\omega=3\%$; $\gamma_d/\omega=2\%$.

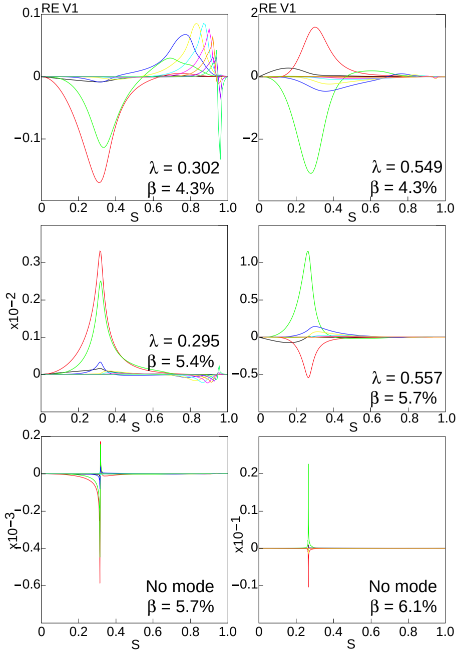
Absolute amplitude of TAE-perturbation could be estimated from the frequency-sweeping rate [7]

[7] S.D.Pinches et al., Plasma Physics Controlled Fusion 46 (2004) S47

HIGHER-BETA DISCHARGES: TAEs AT GROWING PRESSURE



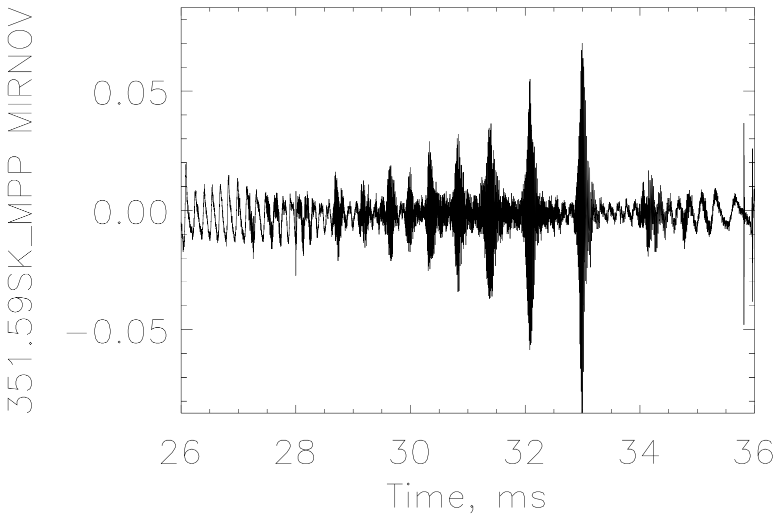
Continuous spectrum of the shear Alfvén waves in START (β=3.9%)



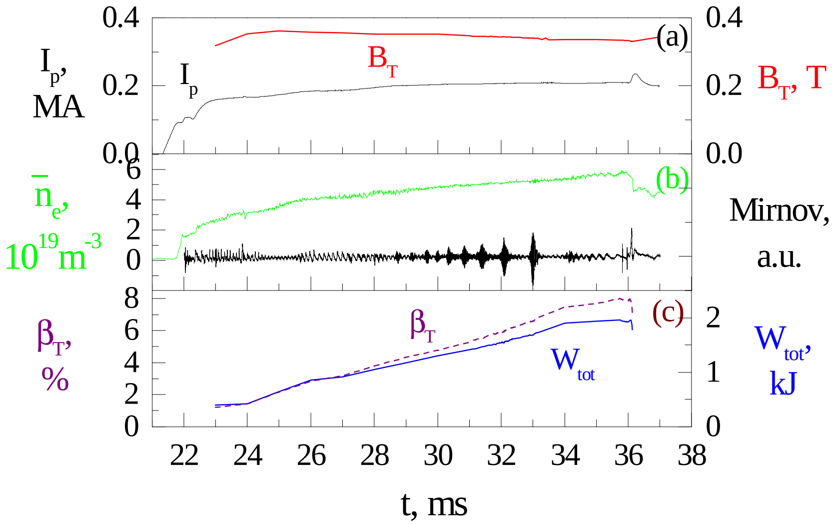
The radial structure of the eigenfunctions of lower (left) and upper core-localised TAE at different values of thermal plasma β. TAE disappears at $\alpha \geq \alpha_{crit} = \varepsilon + 2\Delta' \pm S^2$ (see [8])

[8] M.P.Gryaznevich, S.E.Sharapov, PPCF 46 (2004) S15

CHIRPING MODES IN START DISCHARGES

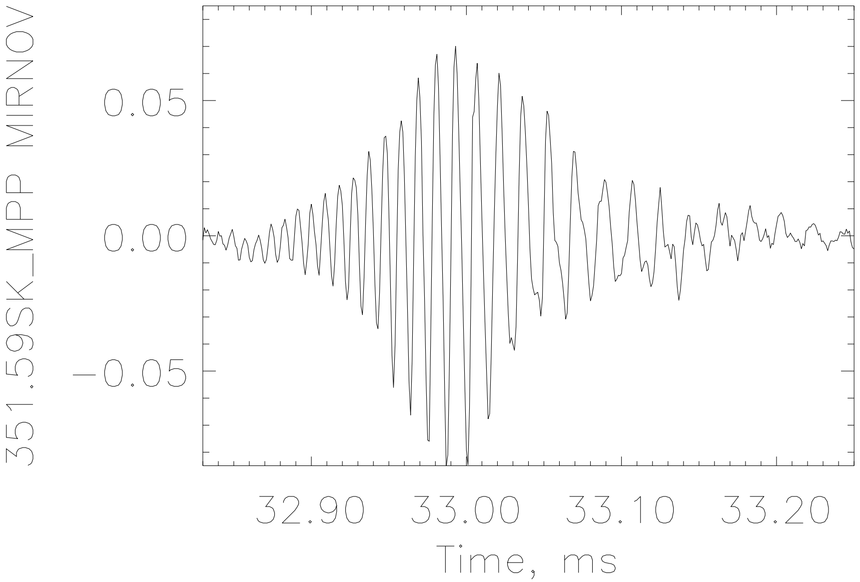


Magnetic perturbations $\partial(\delta B_P)/\partial t$ showing chirping modes detected by the outboard Mirnov coil.

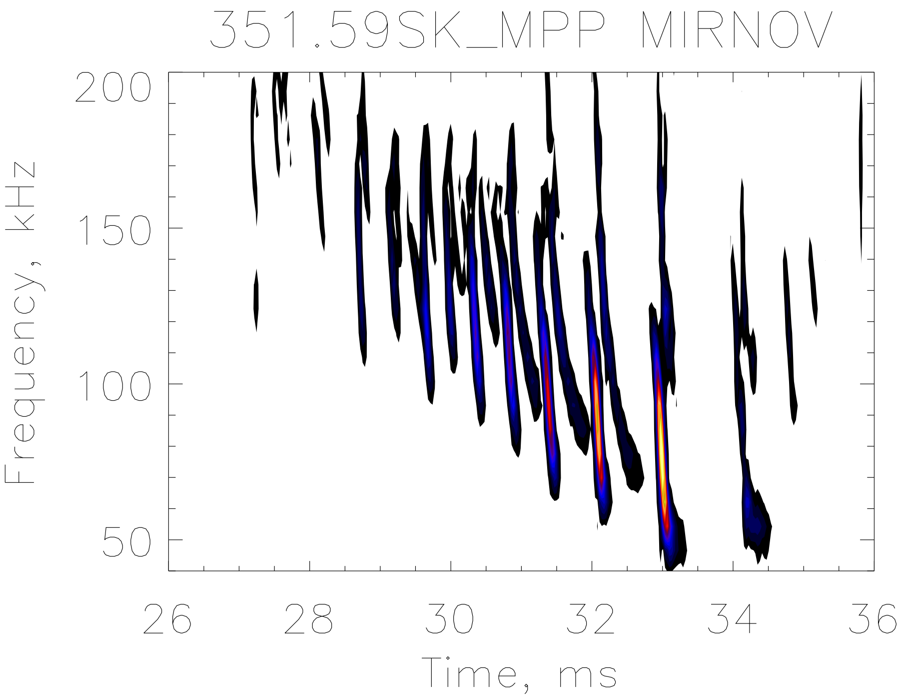


Temporal evolution of plasma current I_P, toroidal magnetic field B_T, line-averaged plasma density, volume-averaged β, plasma energy content, and the bursts of magnetic perturbations in NBI heated START discharge #35159

CHIRPING MODES IN START DISCHARGES: SOFT X-RAY

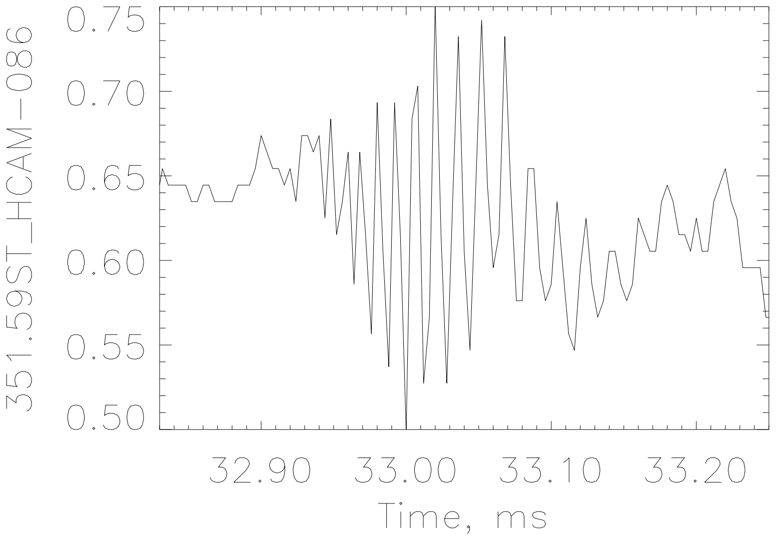


Zoom of a single burst showing frequency-sweeping ‘chirping’ mode on Mirnov coil

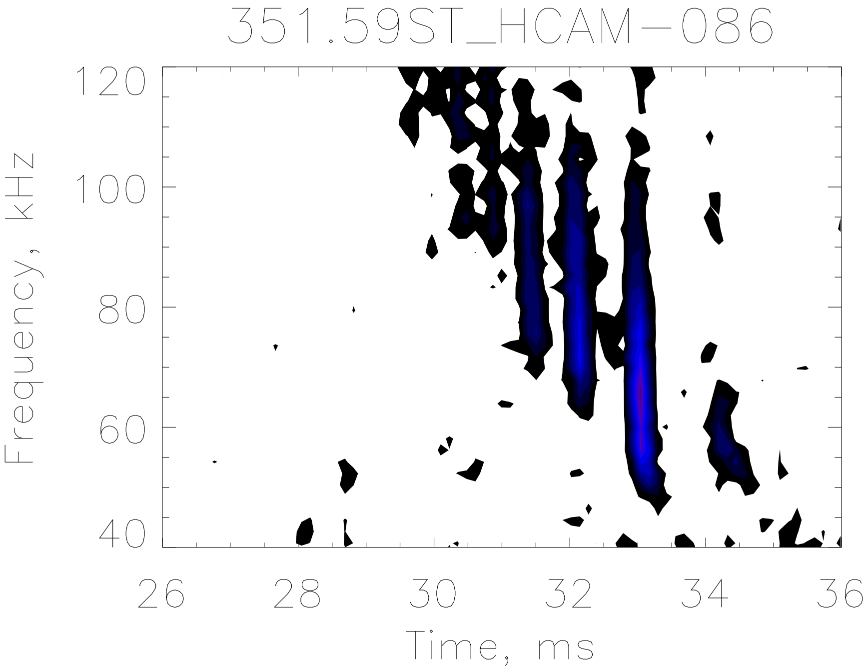


Magnetic spectrogram showing amplitude of chirping modes as function of time and the mode frequency.

CHIRPING MODES IN START DISCHARGES: SOFT X-RAY



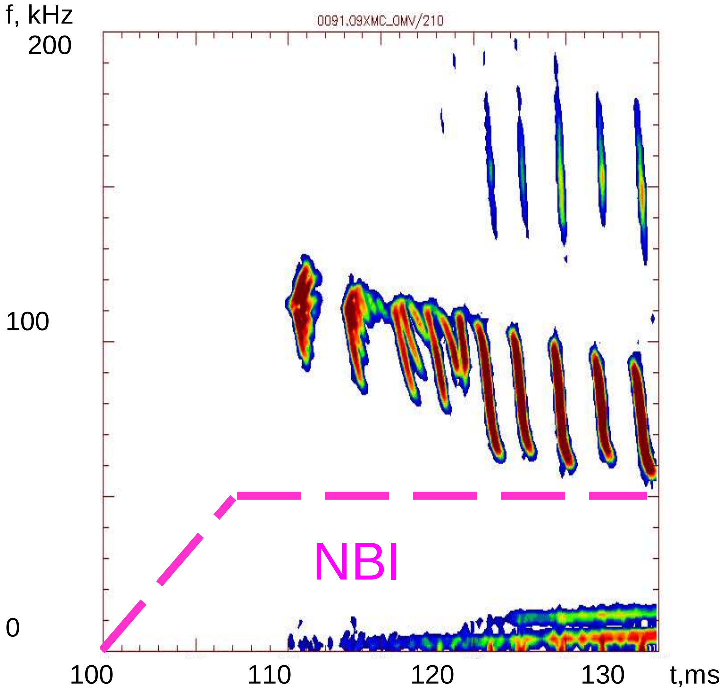
The same frequency-sweeping perturbation seen by the horizontal SXR camera with chord at Z=-6.1 cm.



The Fourier transformed SXR data showing the same chirping modes

S.E.Sharapov, M.P.Gryaznevich et al, 10th ST Workshop, 29 September - 1 October 2004, Kyoto, Japan

CHIRPING MODES IN MAST DISCHARGES



Chirping modes similar to those observed on START, are also typical for MAST (example shows MAST #9109, 1.2 MW of 40 keV NBI at I_P flat-top, β≈3%). New: chirping modes with higher n = 3 observed.

CHIRPING MODES

- **In higher β discharges, i.e. 5% < β < 15%, the Alfvén instabilities on MAST and START were dominated by 'chirping' modes[9]**

- **These modes are identified as non-perturbative EPMs[10-13]. Much larger fractional frequency shift ($\delta\omega / \omega \sim 50\%$) for chirping modes than that for hole-clump pairs ($\delta\omega / \omega \leq 20\%$) show that a non-perturbative EPM triggers larger sweeps than a perturbative TAE similar to the perturbative vs non-perturbative fishbone simulation[14].**

- **How these modes behave as β increases further, to β > 15%? Stronger stabilising effect of thermal ion Landau damping is expected.**

[9] W.W.Heidbrink, PPCF 37 (1995) 937

[10] Liu Chen, Phys. Plasmas 1 (1994) 1519

[11] F.Zonca, L.Chen, Physics of Plasmas 3 (1996) 323

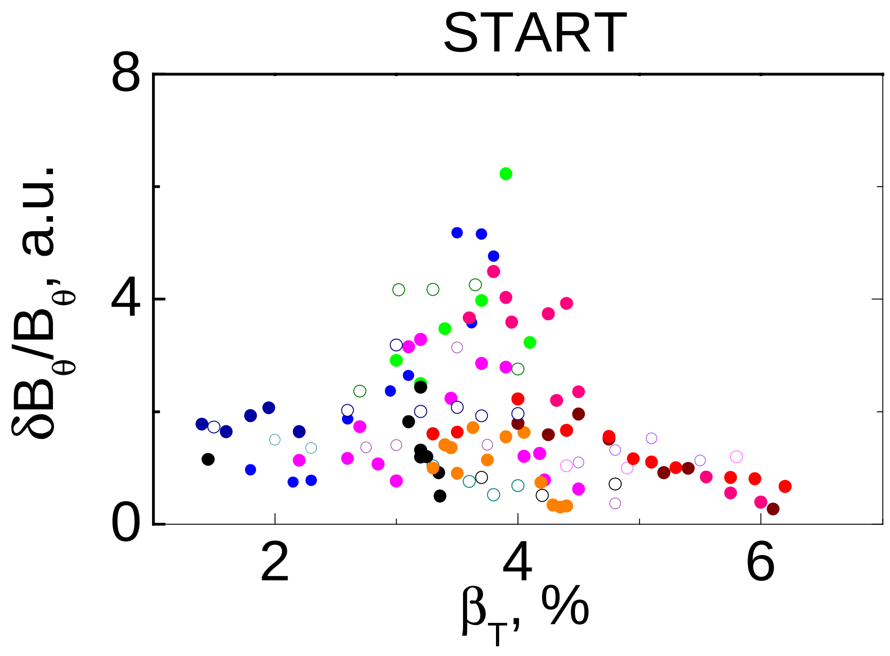
[12] C.Z.Cheng et al., Nuclear Fusion 35 (1995) 1639

[13] M.P.Gryaznevich, S.E.Sharapov, Nuclear Fusion 40 (2000) 907

[14] J.Candy, H.L.Berk, B.N.Breizman, F.Porcelli, Physics of Plasmas 6 (1999) 1822

AMPLITUDE OF CHIRPING MODES AS FUNCTION OF β: START

- **On START, the chirping mode amplitude decreases as beta increases.**
- **No chirping modes observed at beta > 6.5 %.**
- **Initial increase of mode amplitude with beta may be related to increase in the fast ion pressure.**



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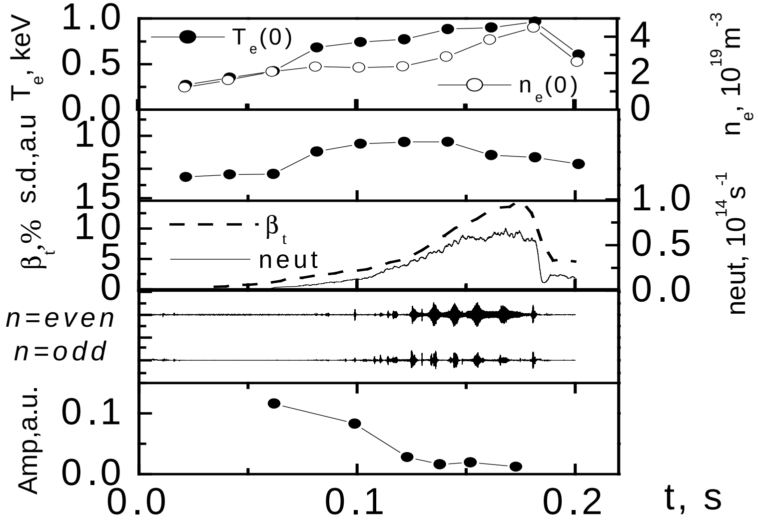
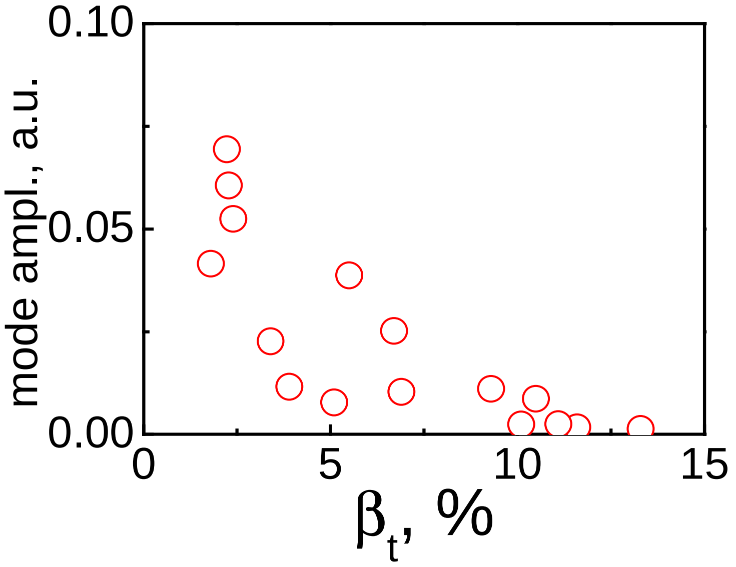
AMPLITUDE OF CHIRPING MODES AS FUNCTION OF β: MAST





Signals for a typical 0.8 MA, 0.45 T MAST discharge #8977 with NBI power 2.7 MW. Amplitude of chirping modes (bottom) decreases with increasing β at nearly constant slowing-down time.

Dependence on β of the maximum amplitude in a single burst of chirping modes, in NBI discharges on MAST

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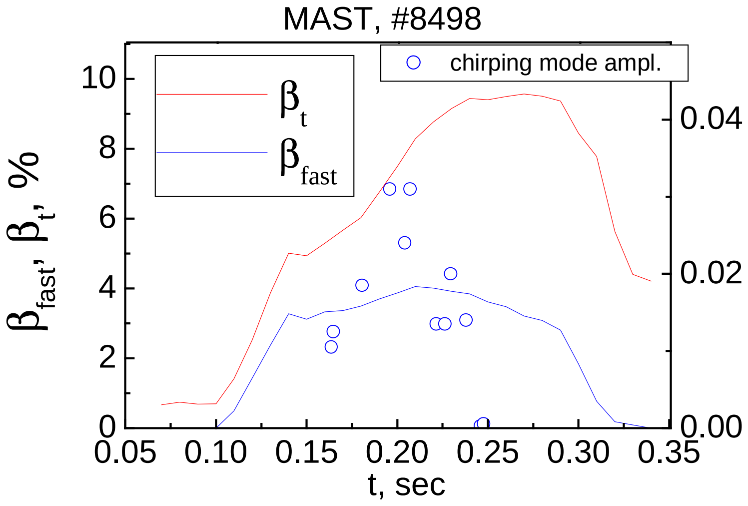
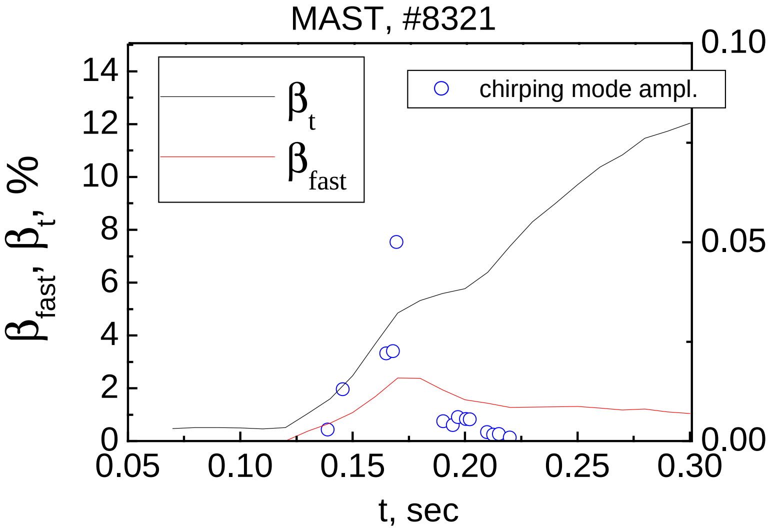
AMPLITUDE OF CHIRPING MODES AS FUNCTION OF β: MAST





TRANSP analysis showing β_{fast} and $\beta_{thermal}$ together with the chirping mode amplitudes in MAST discharge #8498

TRANSP analysis showing β_{fast} and $\beta_{thermal}$ together with the chirping mode amplitudes in MAST discharge #8321

CONCLUSIONS

- STs are a perfect test-bed for studying Alfvén instabilities in a wide range of plasma and fast ion parameters.

- Both perturbative and non-perturbative Alfén Eigenmodes observed.

- Three different regimes of high-frequency Alfvén instabilities in ST:

1) Low-beta “classical” TAE regime;
2) Medium-beta “chirping mode” regime;
3) High-beta, $\beta(0) \approx 1$, regime relevant for burning ST.

- Low-beta regime shows TAEs & EAEs.

- Pitchfork splitting and frequency-sweeping modes emerging from TAEs are observed.

- Modelling with the HAGIS code shows that these sweeping modes can be identified as hole-clump pairs.

- Suppression of TAEs by the pressure effect was investigated. For typical START and MAST data, no TAEs observed at $\beta > 5\%$.

- For chirping modes, a decrease in mode amplitude as beta increases was established for both START and MAST data.

- These findings show that the main Alfvén instabilities driven by gradient of fast ion pressure, TAEs and the chirping modes are likely to be absent in burning plasma STs with $\beta(0) \approx 1$. Remaining known instabilities (fishbones and compressional Alfvén eigenmodes) must be investigated in the high-beta regimes experimentally.